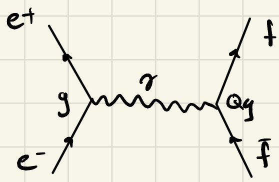
PHY489 Week 9/10: Electrodynamics of Quarks & Hadrons

The process $e^+e^- \to \gamma \to f\bar{f}$ remains the same regardless of what fermion/antifermion pair is in the final state.

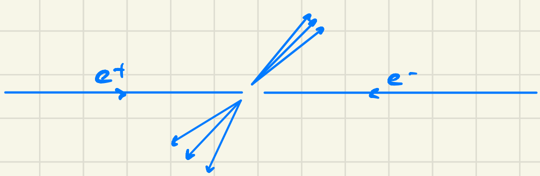


Q is electric charge; $\neq 1$ for quarks!
(addition factor of Q in $\mathcal{M}_{fi}$)

→ Quarks cannot exist freely and must hadronize (these produce "jets" of particles in final states)

High enough energies we assume the scattering process decouples from the hadronization process, that is,

$$\sigma(e^+e^- \to \text{Hadrons}) = \sum_{q=u,d,s,c,b,t} \sigma(e^+e^- \to q\bar{q})$$



We can consider the ratio of the cross sections of muon pair production to that of hadron production:

$$R(E) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

A function of energy, since quarks are only kinematically available depending on the magnitude of the centre of mass energy.

For any $f\bar{f}$ pair of fermions, it's easy to show that

$$\sigma(e^+e^- \to f\bar{f}) = \frac{\pi}{3}\left(\frac{\hbar c\, Q_f \alpha}{E}\right)^2$$

implying that

$$R(E) = 3 \sum_{f=u,d,s,c,b,t} Q_f^2$$

Q_f is charge of quark

3 since there are 3 quark degeneracies due to color

CM Energy Value	Contributing Quarks	R(E) Value
2 GeV	u, d, s	2
4 GeV	u, d, s, c	$\frac{10}{3}$
10 GeV	u, d, s, c, b	$\frac{11}{3}$

Angular Distributions of $e^+e^- \to f\bar{f}$:

We have that $$\frac{d\sigma}{d\Omega}(e^+e^- \to f\bar{f}) = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\langle |\mathcal{M}_{fi}|^2 \rangle}{(2E)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

with $$\langle |\mathcal{M}_{fi}|^2 \rangle = g^4 \underbrace{\left\{ 1 + \frac{(mc^2)^2}{E^2} + \frac{(Mc^2)^2}{E^2} + \left[1 - \left(\frac{mc^2}{E}\right)^2\right]\left[1 - \left(\frac{Mc^2}{E}\right)^2\right] \cos^2\theta \right\}}_{\simeq 1 + \cos^2\theta \text{ in the high-energy limit.}}$$

Implying that final state (spin-$\frac{1}{2}$ fermion) angular distributions should be of the form $1 + \cos^2\theta$ at energies well above the relevant fermion mass.